

On the Lebesgue constant of barycentric rational Hermite interpolants at equidistant nodes

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Abstract

Barycentric rational Floater–Hormann interpolants compare favourably to classical polynomial interpolants in the case of equidistant nodes, because the Lebesgue constant associated with these interpolants grows logarithmically in this setting, in contrast to the exponential growth experienced by polynomials. In the Hermite setting, in which also the first derivatives of the interpolant are prescribed at the nodes, the same exponential growth has been proven for polynomial interpolants, and the main goal of this paper is to show that much better results can be obtained with a recent generalization of Floater–Hormann interpolants. After summarizing the construction of these barycentric rational Hermite interpolants, we study the behaviour of the corresponding Lebesgue constant and prove that it is bounded from above by a constant. Several numerical examples confirm this result.

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1 Introduction

Let $n \in \mathbb{N}$ and $X_n = \{x_0, x_1, \dots, x_n\}$ be a set of $n + 1$ nodes, such that $0 = x_0 < x_1 < \dots < x_n = 1$. Given two sets of basis functions $b_{0,i}$ and $b_{1,i}$, $i = 0, \dots, n$, satisfying

$$\begin{aligned} b_{0,i}(x_j) &= \delta_{i,j}, & b'_{0,i}(x_j) &= 0, \\ b_{1,i}(x_j) &= 0, & b'_{1,i}(x_j) &= \delta_{i,j}, \end{aligned}$$

for $j = 0, \dots, n$, and some $f \in C^1[0, 1]$, we are interested in the class of linear Hermite interpolants $h_n : [0, 1] \rightarrow \mathbb{R}$ of f of the form

$$h_n(x) = \sum_{i=0}^n b_{0,i}(x) f_i + \sum_{i=0}^n b_{1,i}(x) f'_i, \quad (1)$$

where

$$f_i = f(x_i), \quad f'_i = f'(x_i).$$

An example is given by the classical polynomial Hermite interpolants [5, 9], where the basis functions are defined as

$$b_{0,i}(x) = (1 - 2(x - x_i)\ell'_i(x_i))\ell_i(x)^2, \quad b_{1,i}(x) = (x - x_i)\ell_i(x)^2, \quad (2)$$

with ℓ_i denoting the usual Lagrange basis polynomials

$$\ell_i(x) = \prod_{\substack{j=0 \\ j \neq i}}^n \frac{x - x_j}{x_i - x_j}.$$

Extending the concept of the Lebesgue constant from Lagrange to Hermite interpolation, we call

$$\Lambda_n = \sup_{\|f\|_1=1} \|h_n\|$$

the Lebesgue constant of the Hermite interpolant (1), where $\|\cdot\|$ is the supremum norm in $C[0, 1]$ and

$$\|f\|_1 = \|f\| + \|f'\|.$$

It is then easy to verify [6] that

$$\Omega_{1,n} \leq \Lambda_n \leq \max\{\Omega_{0,n}, \Omega_{1,n}\}, \quad (3)$$

where

$$\Omega_{0,n} = \max_{0 \leq x \leq 1} \Omega_{0,n}(x), \quad \Omega_{1,n} = \max_{0 \leq x \leq 1} \Omega_{1,n}(x). \quad (4)$$

and

$$\Omega_{0,n}(x) = \sum_{i=0}^n |b_{0,i}(x)|, \quad \Omega_{1,n}(x) = \sum_{i=0}^n |b_{1,i}(x)|. \quad (5)$$

Both $\Omega_{0,n}$ and $\Omega_{1,n}$ play a crucial role in measuring the approximation quality of h_n , since [7]

$$\|f - h_n\| \leq (1 + \Omega_{0,n} + \Omega_{1,n}) \|f' - p_{2n}\|,$$

where p_{2n} is the polynomial of degree at most $2n$ that approximates f' best on $[0, 1]$, and have therefore been investigated intensively. For example, it is known that

$$\Omega_{0,n} = 1, \quad \Omega_{1,n} \leq \frac{1}{\rho}, \quad (6)$$

for ρ -normal sets of nodes [7], which implies convergence of the Hermite interpolation process. Examples of such sets are given by the roots of the Jacobi polynomials $J_n^{(\alpha,\beta)}$ which are normal for $\alpha, \beta \leq 0$ and ρ -normal for $\alpha, \beta < 0$, with $\rho = \min\{-\alpha, -\beta\}$ [7]. Chebyshev nodes are a special case of $\frac{1}{2}$ -normal nodes, since they are the zeros of the Chebyshev polynomials $J_n^{(-\frac{1}{2}, -\frac{1}{2})}$. For Chebyshev nodes, the upper bound in (6) can be improved significantly [10] to

$$\Omega_{1,n} \leq \frac{\log n}{n} C,$$

where C is a constant independent of n . Other results about polynomial Hermite interpolation for normal and ρ -normal sets of nodes can be found in [11] and references therein.

However, this favourable behaviour does not hold in another common interpolation setting, the equidistant case with nodes

$$x_i = \frac{i}{n},$$

for $i = 0, \dots, n$. Indeed, both $\Omega_{0,n}$ and $\Omega_{1,n}$ grow very fast as $n \rightarrow \infty$, namely [6]

$$\Omega_{0,n} \sim \frac{2^{2n+1}}{\gamma_n^2 n^2} C, \quad \Omega_{1,n} \sim \frac{2^{2n+1}}{\gamma_n^2 n^2 \sqrt{n}} C,$$

where $\gamma_n = \sum_{j=1}^n 1/j$. This unfavourable growth also reflects the ill-conditioning of polynomial Hermite interpolation at equidistant nodes, which may lead to wild oscillations of h_n , just as in the Lagrange interpolation case.

In the Lagrange case, barycentric rational Floater–Hormann interpolants [4] have been shown to have a much better conditioning than polynomial interpolants at equidistant nodes, since the related Lebesgue constants grow only logarithmically with n and exponentially with d , a parameter related to the construction and the approximation order of the Floater–Hormann interpolants [1, 2]. In this paper we show that this favourable behaviour also holds for a generalization of these interpolants to the Hermite setting. After reviewing Cirillo and Hormann's [3] iterative construction of barycentric rational Hermite interpolants (Section 2), we show that for the special case of Floater–Hormann Hermite interpolation both $\Omega_{0,n}$ and $\Omega_{1,n}$ can be bounded from above by a constant that grows exponentially with d (Section 3). We conclude with some numerical examples that confirm this result (Section 4).

2 Iterative barycentric rational Hermite interpolation

Schneider and Werner [8] show that the rational function

$$r_n(x) = \sum_{i=0}^n b_i(x) f_i, \quad (7)$$

with basis functions

$$b_i(x) = \frac{w_i}{x - x_i} \bigg/ \sum_{k=0}^n \frac{w_k}{x - x_k} \quad (8)$$

interpolates the data f_j at x_j for $j = 0, \dots, n$, as long as all the weights w_i are non-zero. It is clear that these b_i satisfy the Lagrange property

$$b_i(x_j) = \delta_{i,j} \quad (9)$$

and form a partition of unity,

$$\sum_{i=0}^n b_i(x) = 1, \quad (10)$$

just like the Lagrange basis polynomials ℓ_i , and we recall from [8, Proposition 11] that

$$b'_i(x_i) = \sum_{\substack{j=0 \\ j \neq i}}^n \frac{w_j}{w_i(x_j - x_i)}, \quad b'_i(x_j) = \frac{w_i}{w_j(x_j - x_i)}, \quad j \neq i. \quad (11)$$

Cirillo and Hormann [3] show that these interpolants can be extended to the Hermite setting by letting

$$h_n(x) = r_n(x) + q_n(x), \quad (12)$$

where the correction term

$$q_n(x) = \sum_{i=0}^n (x - x_i) b_i(x)^2 (f'_i - r'_n(x_i))$$

is chosen so as to fix the interpolation of the derivative data f'_i without altering the interpolation of the data f_i . For our purposes, it turns out to be useful to rewrite the barycentric rational Hermite interpolant h_n in (12) in the form (1) with $b_{0,i}$ and $b_{1,i}$ as in (2), but with ℓ_i replaced by b_i in (8).

Proposition 2.1. *The barycentric rational Hermite interpolant h_n in (12) can be written as*

$$h_n(x) = \sum_{i=0}^n b_{0,i}(x) f_i + \sum_{i=0}^n b_{1,i}(x) f'_i \quad (13)$$

with

$$b_{0,i}(x) = (1 - 2(x - x_i) b'_i(x_i)) b_i(x)^2, \quad b_{1,i}(x) = (x - x_i) b_i(x)^2.$$

Proof. By (12) and (7),

$$h_n(x) = \sum_{i=0}^n b_i(x) f_i + \sum_{i=0}^n (x - x_i) b_i(x)^2 f'_i - \sum_{i=0}^n (x - x_i) b_i(x)^2 \sum_{j=0}^n b'_j(x_i) f_j.$$

It remains to show that

$$b_i(x) - \sum_{j=0}^n (x - x_j) b_j(x)^2 b'_i(x_j) = b_i(x)^2 - 2(x - x_i) b'_i(x_i) b_i(x)^2. \quad (14)$$

Using (10) and (11) we have

$$\begin{aligned} b_i(x) - \sum_{j=0}^n (x - x_j) b_j(x)^2 b'_i(x_j) &= b_i(x)^2 + \sum_{\substack{j=0 \\ j \neq i}}^n b_i(x) b_j(x) - (x - x_i) b_i(x)^2 b'_i(x_i) - \sum_{\substack{j=0 \\ j \neq i}}^n (x - x_j) b_j(x)^2 \frac{w_i}{w_j(x_j - x_i)} \\ &= b_i(x)^2 - (x - x_i) b_i(x)^2 b'_i(x_i) + \sum_{\substack{j=0 \\ j \neq i}}^n \left(b_i(x) - \frac{x - x_j}{x_j - x_i} b_j(x) \frac{w_i}{w_j} \right) b_j(x). \end{aligned}$$

Now, since

$$b_j(x) = \frac{x - x_i}{x - x_j} b_i(x) \frac{w_j}{w_i}$$

by (8), we find that

$$\left(b_i(x) - \frac{x - x_j}{x_j - x_i} b_j(x) \frac{w_i}{w_j} \right) b_j(x) = \left(1 - \frac{x - x_i}{x_j - x_i} \right) b_i(x) b_j(x) = -(x - x_i) b_i(x)^2 \frac{w_j}{w_i(x_j - x_i)},$$

and (14) then follows by using again (11). \square

3 Floater–Hormann Hermite interpolation

Floater and Hormann [4] show that for any d with $0 \leq d \leq n$, the special choice of barycentric weights

$$w_i = (-1)^i v_i$$

with

$$v_i = \sum_{j=\max\{0, i-d\}}^{\min\{i, n-d\}} \prod_{\substack{k=j \\ k \neq i}}^{j+d} \frac{1}{|x_i - x_k|} > 0 \quad (15)$$

in (8) gives barycentric rational interpolants without any real poles and approximation order $O(h^{d+1})$ for sufficiently smooth functions. As shown in [2], in the special case of equidistant nodes, the corresponding Lebesgue constant grows logarithmically in the number of nodes. For $n \geq 2d$ equidistant nodes, the values v_i in (15) can be simplified to (cf. [2])

$$v_i = \sum_{j=d}^n \binom{d}{j-i} \leq 2^d, \quad (16)$$

and we shall now derive an upper bound of the Lebesgue constant associated with the barycentric rational Hermite interpolant h_n in (13) for this choice of v_i .

More precisely, we derive upper bounds for $\Omega_{0,n}$ and $\Omega_{1,n}$ in (4) and then use (3). Inspired by the proof of Theorem 1 in [2], we first focus on the case where $x_k < x < x_{k+1}$ for some k with $0 \leq k \leq n-1$ and rewrite $\Omega_{0,n}(x)$ and $\Omega_{1,n}(x)$ as

$$\Omega_{0,n}(x) = \frac{N_{0,k}(x)}{D_k(x)}, \quad \Omega_{1,n}(x) = \frac{N_{1,k}(x)}{D_k(x)},$$

where

$$N_{0,k}(x) = (x - x_k)^2 (x_{k+1} - x)^2 \sum_{i=0}^n |1 - 2(x - x_i) b'_i(x_i)| \frac{v_i^2}{(x - x_i)^2}, \quad (17)$$

$$N_{1,k}(x) = (x - x_k)^2 (x_{k+1} - x)^2 \sum_{i=0}^n \frac{v_i^2}{|x - x_i|}, \quad (18)$$

and

$$D_k(x) = (x - x_k)^2 (x_{k+1} - x)^2 \left(\sum_{i=0}^n \frac{w_i}{x - x_i} \right)^2.$$

As proved in [2], the denominator satisfies

$$D_k(x) \geq \frac{1}{n^2}, \quad (19)$$

and it remains to establish appropriate upper bounds for the numerators $N_{0,k}(x)$ and $N_{1,k}(x)$.

Lemma 3.1. *Let $n \geq 2d$ and $x_k < x < x_{k+1}$ for some k with $0 \leq k \leq n-1$. Then,*

$$N_{1,k}(x) \leq \frac{4^d}{n^2} C,$$

for some constant C that does not depend on k , d , and n .

Proof. Since

$$\sum_{i=0}^n \frac{v_i^2}{|x - x_i|} = \sum_{i=0}^k \frac{v_i^2}{x - x_i} + \sum_{i=k+1}^n \frac{v_i^2}{x_i - x} \leq \sum_{i=0}^k \frac{v_i^2}{x - x_k} + \sum_{i=k+1}^n \frac{v_i^2}{x_{k+1} - x}.$$

and

$$(x - x_k)(x_{k+1} - x)^2 \leq \frac{4}{27n^3}, \quad (x - x_k)^2 (x_{k+1} - x) \leq \frac{4}{27n^3}, \quad (20)$$

we have

$$N_{1,k}(x) \leq \frac{4}{27n^3} \sum_{i=0}^n v_i^2,$$

and the statement then follows from (16). \square

Lemma 3.2. Let $n \geq 2d$ and $x_k < x < x_{k+1}$ for some k with $0 \leq k \leq n-1$. Then,

$$N_{0,k}(x) \leq \frac{4^d(d+1)}{n^2} C,$$

for some constant C that does not depend on k , d , and n .

Proof. Using (11) and (16), we first notice that

$$\sum_{i=0}^n |1 - 2(x - x_i)b'_i(x_i)| \frac{v_i^2}{(x - x_i)^2} \leq 4^d \sum_{i=0}^n \frac{1}{(x - x_i)^2} + 2^{d+1} \sum_{i=0}^n \frac{1}{|x - x_i|} \left| \sum_{\substack{j=0 \\ j \neq i}}^n \frac{(-1)^j v_j}{x_j - x_i} \right|,$$

and we proceed to bound the two sums over i separately. For $x_k < x < x_{k+1}$, we have

$$\begin{aligned} \sum_{i=0}^n \frac{1}{(x - x_i)^2} &= \sum_{i=0}^{k-1} \frac{1}{(x - x_i)^2} + \frac{1}{(x - x_k)^2} + \frac{1}{(x_{k+1} - x)^2} + \sum_{i=k+2}^n \frac{1}{(x_i - x)^2} \\ &\leq \sum_{i=0}^{k-1} \frac{1}{(x_k - x_i)^2} + \frac{(x_{k+1} - x_k)^2}{(x - x_k)^2 (x_{k+1} - x)^2} + \sum_{i=k+2}^n \frac{1}{(x_i - x_{k+1})^2} \\ &= \sum_{i=0}^{k-1} \frac{n^2}{(k-i)^2} + \frac{1}{n^2} \frac{1}{(x - x_k)^2 (x_{k+1} - x)^2} + \sum_{i=k+2}^n \frac{n^2}{(i-k-1)^2} \\ &= n^2 \sum_{i=1}^k \frac{1}{i^2} + \frac{1}{n^2} \frac{1}{(x - x_k)^2 (x_{k+1} - x)^2} + n^2 \sum_{i=1}^{n-k-1} \frac{1}{i^2} \\ &\leq n^2 \frac{\pi^2}{6} + \frac{1}{n^2} \frac{1}{(x - x_k)^2 (x_{k+1} - x)^2} + n^2 \frac{\pi^2}{6}, \end{aligned}$$

and since

$$(x - x_k)^2 (x_{k+1} - x)^2 \leq \frac{1}{16n^4}, \quad (21)$$

we conclude that

$$(x - x_k)^2 (x_{k+1} - x)^2 \sum_{i=0}^n \frac{1}{(x - x_i)^2} \leq \frac{1}{n^2} C.$$

To bound the second sum, we first use (16) to get

$$\left| \sum_{\substack{j=0 \\ j \neq i}}^n \frac{(-1)^j v_j}{j - i} \right| = \left| \sum_{j=0}^n \frac{(-1)^j}{j - i} \sum_{l=d}^n \binom{d}{l-j} \right| = \left| \sum_{l=0}^d \binom{d}{l} \sum_{\substack{j=d-l \\ j \neq i}}^{n-l} \frac{(-1)^j}{j - i} \right| \leq 2^d \max_{0 \leq l \leq d} \left| \sum_{\substack{j=d-l \\ j \neq i}}^{n-l} \frac{(-1)^j}{j - i} \right|,$$

and since

$$\left| \sum_{\substack{j=d-l \\ j \neq i}}^{n-l} \frac{(-1)^j}{j - i} \right| = \begin{cases} \left| \sum_{j=d-l}^{n-l} \frac{(-1)^j}{j - i} \right| \leq \frac{1}{(d-l) - i}, & 0 \leq i < d-l, \\ \left| \sum_{j=1}^{i-(d-l)} \frac{(-1)^j}{j} - \sum_{j=1}^{(n-l)-i} \frac{(-1)^j}{j} \right| \leq \begin{cases} \frac{1}{i - (d-l) + 1}, & d-l \leq i \leq \frac{n+d}{2} - l, \\ \frac{1}{(n-l) - i + 1}, & \frac{n+d}{2} - l \leq i \leq n-l, \end{cases} \\ \left| \sum_{j=d-l}^{n-l} \frac{(-1)^j}{i - j} \right| \leq \frac{1}{i - (n-l)}, & n-l < i \leq n, \end{cases}$$

we further have

$$c_i = \left| \sum_{\substack{j=0 \\ j \neq i}}^n \frac{(-1)^j v_j}{j - i} \right| \leq 2^d \begin{cases} 1, & 0 \leq i \leq d, \\ \frac{1}{i - d + 1}, & d \leq i \leq \frac{n}{2}, \\ \frac{1}{(n-d) - i + 1}, & \frac{n}{2} \leq i \leq n-d, \\ 1, & n-d \leq i \leq n. \end{cases}$$

Let us now assume that $d \leq k < n/2 - 1$ and $x_k < x < x_{k+1}$. Then,

$$\sum_{i=0}^{k-1} \frac{c_i}{k-i} \leq 2^d \left(\sum_{i=0}^{d-1} \frac{1}{k-i} + \sum_{i=d}^{k-1} \frac{1}{(k-i)(i-d+1)} \right) \leq 2^d(d+1),$$

and

$$\frac{c_k}{x-x_k} \leq \frac{2^d}{x-x_k}, \quad \frac{c_{k+1}}{x_{k+1}-x} \leq \frac{2^d}{x_{k+1}-x},$$

and

$$\begin{aligned} \sum_{i=k+2}^n \frac{c_i}{i-k-1} &\leq 2^d \left(\sum_{i=k+2}^{\lfloor n/2 \rfloor} \frac{1}{i-k-1} \left(\frac{1}{i-d+1} - \frac{1}{n-d-i+1} \right) \right. \\ &\quad \left. + \sum_{i=k+2}^{n-d} \frac{1}{(i-k-1)(n-d-i+1)} + \sum_{i=n-d+1}^n \frac{1}{i-k-1} \right) \\ &\leq 2^d \left(\sum_{i=k+2}^{\lfloor n/2 \rfloor} \frac{1}{(i-k-1)(i-d+1)} + \sum_{i=1}^{n-d-k-1} \frac{1}{i(n-d-k-i)} + \sum_{i=n-d}^{n-1} \frac{1}{i-k} \right) \\ &\leq 2^d \left(\frac{\pi^2}{6} + 1 + d \right). \end{aligned}$$

Therefore,

$$\begin{aligned} \sum_{i=0}^n \frac{1}{|x-x_i|} \left| \sum_{\substack{j=0 \\ j \neq i}}^n \frac{(-1)^j v_j}{x_j - x_i} \right| &\leq n^2 \sum_{i=0}^{k-1} \frac{c_i}{k-i} + n \frac{c_k}{x-x_k} + n \frac{c_{k+1}}{x_{k+1}-x} + n^2 \sum_{i=k+2}^n \frac{c_i}{i-k-1} \\ &\leq 2^d \left(n^2(d+1) + \frac{n}{x-x_k} + \frac{n}{x_{k+1}-x} + n^2 \left(\frac{\pi^2}{6} + 1 + d \right) \right). \end{aligned}$$

Using (20) and (21), we finally obtain

$$(x-x_k)^2(x_{k+1}-x)^2 \sum_{i=0}^n \frac{1}{|x-x_i|} \left| \sum_{\substack{j=0 \\ j \neq i}}^n \frac{(-1)^j v_j}{x_j - x_i} \right| \leq \frac{2^d(d+1)}{n^2} C.$$

The other cases $k < d$ and $k \geq n/2 - 1$ can be treated similarly. □

We are now ready to state our main result.

Theorem 3.3. *Provided that $n \geq 2d$, the Lebesgue constant associated with Floater–Hormann Hermite interpolation at equidistant nodes satisfies*

$$\Lambda_n \leq 4^d(d+1)C,$$

for some constant C that does not depend on d and n .

Proof. If $x = x_k$ for $k = 0, \dots, n$, then

$$b_{0,i}(x) = (1 - 2(x_k - x_i)b'_i(x_i))b_i(x_k)^2 = \delta_{i,k}, \quad b_{1,i}(x) = (x_k - x_i)b_i(x_k)^2 = 0$$

and consequently $\Omega_{0,n}(x) = 1$ and $\Omega_{1,n}(x) = 0$. Otherwise, it follows from (19), Lemma 3.1, and Lemma 3.2, that there exists some constant C that does not depend on n and d , such that

$$\Omega_{0,n}(x) \leq 4^d(d+1)C, \quad \Omega_{1,n}(x) \leq 4^d C.$$

The statement then follows from (3). □

Note that while the constant C in Theorem 3.3 is independent of d , the upper bound grows exponentially in d .

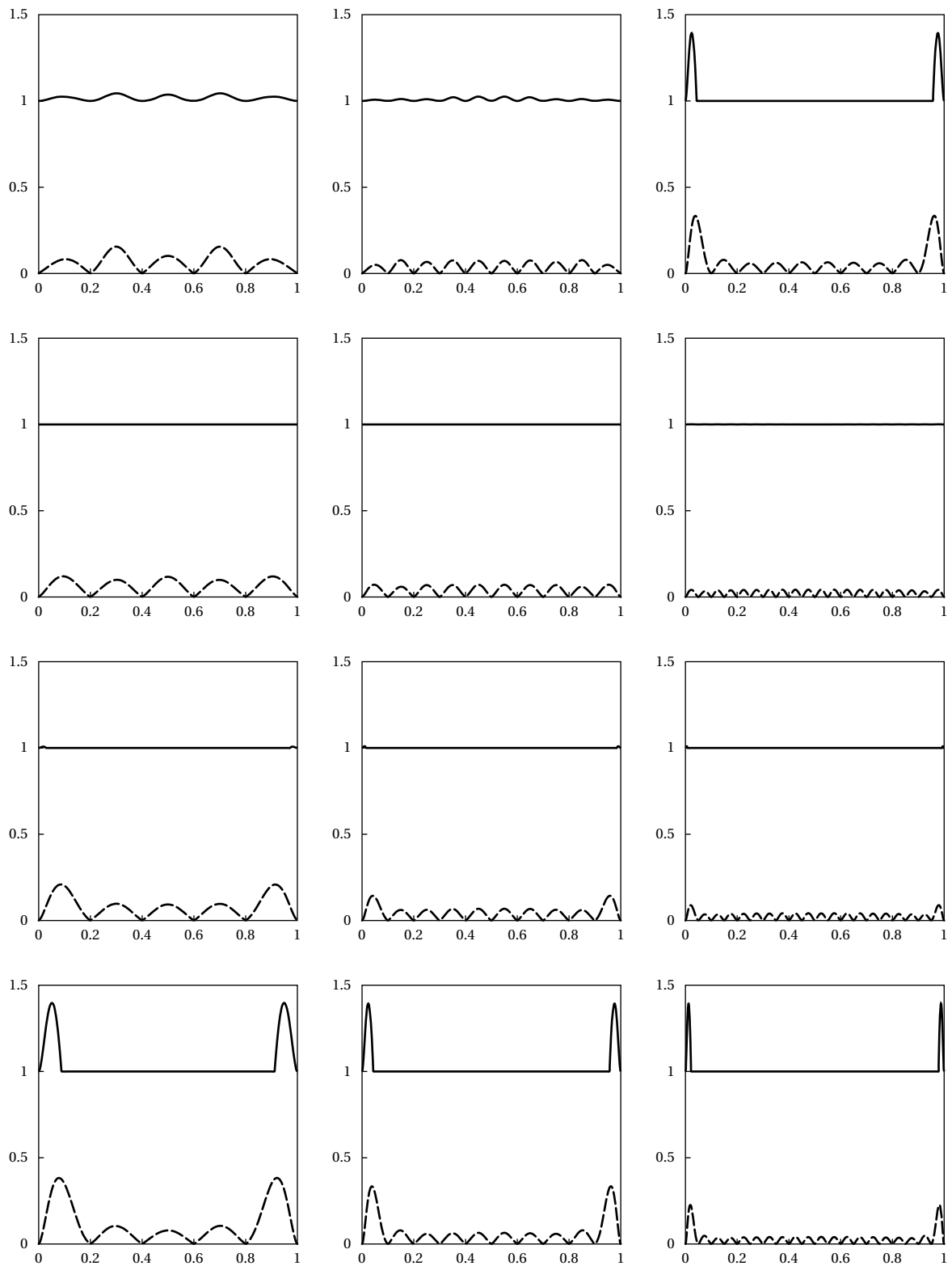


Figure 1: Plot of $\Omega_{0,n}(x)$ (solid line) and $\Omega_{1,n}(x)$ (dashed line) for $d = 0, 1, 2, 3$ (from top to bottom) and $n = 5, 10, 20$ (from left to right) and equidistant nodes in the interval $[0, 1]$.

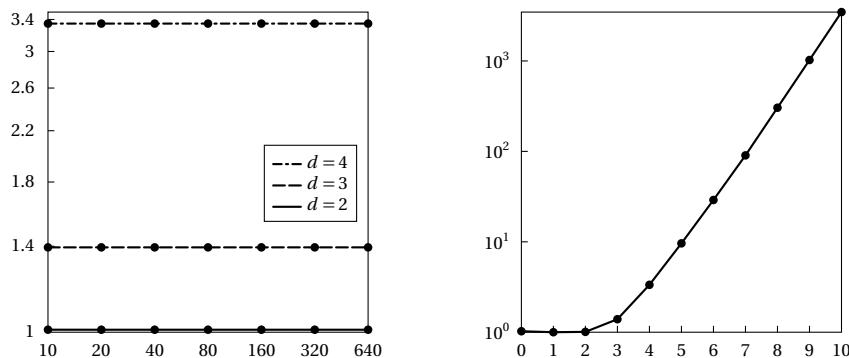


Figure 2: Log-log plot of $\Omega_{0,n}$ over n for different values of d (left) and semi-log plot of $\Omega_{0,n}$ over d for $n = 20$ (right).

4 Numerical results

We performed several experiments to confirm numerically that the upper bounds derived above are correct. Figure 1 shows $\Omega_{0,n}(x)$ and $\Omega_{1,n}(x)$ for Floater–Hormann Hermite interpolation at equidistant nodes in the interval $[0, 1]$ for several values of d and n . Note that $\Omega_{0,n}(x)$ dominates $\Omega_{1,n}(x)$ in all examples, a behaviour that we consistently observed in our experiments. Also note that the maxima $\Omega_{0,n}$ and $\Omega_{1,n}$ of both functions are obtained inside the first and the last sub-interval, except for $d = 0$, and that $\Omega_{0,n}$ is basically independent of n in all examples. This is confirmed by the plot in Figure 2 (left), which additionally shows that $\Omega_{0,n}$, although independent of n , seems to grow exponentially with d , as suggested by the upper bound in Lemma 3.2. This trend can also be observed in Figure 2 (right), where the same quantity is plotted for a fixed value of n and d between 0 and $n/2$.

A completely different result can be observed for non-equidistant nodes. For example, in the case of Chebyshev nodes, $\Omega_{0,n}$ grows quickly as n increases, except for $d = 0$, as shown in Figure 3. We therefore recommend to use Floater–Hormann Hermite interpolation for equidistant nodes, but to stick to polynomial Hermite interpolants for Chebyshev nodes. It remains future work to investigate other choices of interpolation nodes.

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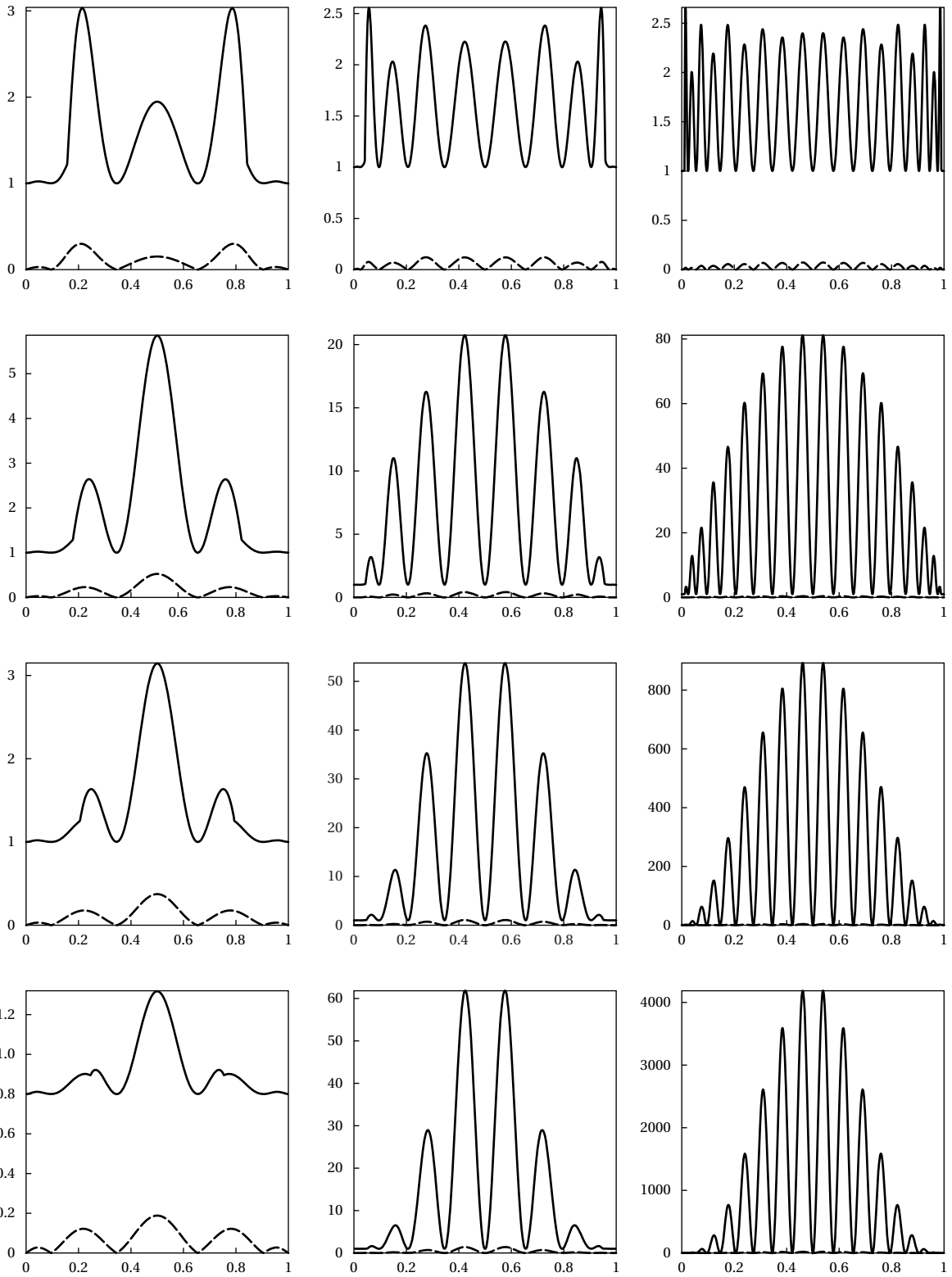


Figure 3: Plot of $\Omega_{0,n}(x)$ (solid line) and $\Omega_{1,n}(x)$ (dashed line) for $d = 0, 1, 2, 3$ (from top to bottom) and $n = 5, 10, 20$ (from left to right) and Chebyshev nodes in the interval $[0, 1]$.